Quantum Vacuum Radiation and Detection Proposals

Haret C. Rosu[1]

We first review the 20-year-old results of Letaw on stationary vacuum radiation patterns originating from world lines defined as Frenet–Serret curves. The corresponding body of literature as well as the experimental proposals that have been suggested to detect quantum vacuum field radiation patterns, are shortly presented and some related topics, such as the anomalous Doppler effect and the decay of accelerated protons are also included.

1. INTRODUCTION

We present a review on vacuum quantum field radiation and related analog radiation patterns organized in three parts that can be considered an update and change-of-viewpoint version of our previous reviews of these topics (Rosu, 2001, 2002). The first part is theoretical and describes this type of radiation as correlated zero-point fluctuations detected by quantum particles forced to move on Frenet–Serret four-dimensional classical trajectories. The second part is an overview of various detection proposals, of which none has turned into a dedicated experiment till now. Moreover, many (almost all) of these proposals bring under the same label different types of quantum field effects being a supply of continuous confusions for many readers. Some of them employ concepts like Unruh radiation, which is most often incorrect since they refer in fact to Unruh's interpretation of the field vacuum as a thermal bath from the point of view of noninertial observers (reference frames) and, if the radiation is real, it should be explained first in the inertial frames. Other experimental setups are closer to interpretations in terms of dynamical (nonadiabatic) Casimir effect. On the other hand, if the case of Frenet–Serret trajectories for relativistic particles is taken into account, then we have real

[1] Department of Applied Mathematics, Potosinian Institute of Science and Technology, Apdo Postal 3-74 Tangamanga, 78231 San Luis Potosi, SLP, Mexico; e-mail: hcr@ipicyt.edu.mx.

vacuum radiation patterns, i.e., in the inertial frame. Thus, the claim to detect is not for Unruh radiation but for signals of the vacuum radiation corresponding to some Frenet–Serret relativistic trajectories of relativistic two-level quantum systems or moving mirrors. Moreover, the radiation patterns will not be exactly of the Frenet–Serret (or Letaw) type that require continuous trajectories up to the third derivative (this can be seen from the tetrad in Eqs. (9–12) below) whereas quantum trajectories are known to be some sort of fractal curves (Sornette, 1990; Abbott and Wise, 1981; Kröger, 1997; Ghose and Home, 1994; Jadczyk, 1995). Nevertheless, the possibility of getting approximate Frenet–Serret radiative vacuum signals is interesting from the fundamental point of view and stimulating for scientific-related technology. Finally, the third part of the present review refers to some related topics such as the anomalous Doppler effect and the decay of accelerated protons.

2. FRENET–SERRET VACUUM RADIATION: THEORY

Frenet (1816–1900) wrote a doctoral thesis in 1847. Part of it deals with the theory of space curves and contains six formulas of the nine three-dimensional Frenet–Serret formulas. He published his results in *Journal de mathematiques pures et appliques* in 1852. Serret (1819–1885) gave the set of all nine formulas in three dimensions.

The standard FS formulas are the following

$$\dot{\vec{T}} = \kappa \cdot \vec{N}$$
$$\dot{\vec{N}} = \tau \cdot \vec{B} - \kappa \cdot \vec{T}$$
$$\dot{\vec{B}} = -\tau \cdot \vec{N}.$$

These three formulas give the derivatives of the unit tangent, normal, and binormal vectors, respectively, as vectorial combinations of the moving basis in which the two Frenet–Serret invariants of the normal three-dimensional space, curvature and torsion, respectively, enter as scalar coefficients. They can be written in matrix form as well

$$\begin{pmatrix} \dot{\vec{T}} \\ \dot{\vec{N}} \\ \dot{\vec{B}} \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} \vec{T} \\ \vec{N} \\ \vec{B} \end{pmatrix}. \tag{1}$$

The plane formed by the span of $\vec{T}$ and $\vec{N}$ is called the osculating plane. The span of $\vec{N}$ and $\vec{B}$ is the normal plane and the span of $\vec{B}$ and $\vec{T}$ is the transverse plane to the curve at the point where the Frenet–Serret triad is considered.

For the goals of the first part of this review, the interested reader may consult several papers of more recent times. These are seminal papers that apply the

Frenet–Serret formalism to relativistic world lines and quantum field theory. First, as quoted by Synge, in 1947, Jasper (1947), studied helices in flat space of n dimensions, but only for a positive–definite metric. Next, Synge wrote an important paper on time-like helices in flat space-time and classified them in six types according to the relative magnitudes of the three Frenet–Serret invariants (Synge, 1967). He also proved for the first time that the world line of a charged particle moving in a constant electromagnetic field is a helix. A few years later, in the seminal paper of Honig *et al.* (1974), a connection was established between the Lorentz invariants of the elctromagnetic field and the Frenet–Serret invariants. The 1981 Letaw's paper (Letaw, 1981), is in fact the first work in which the FS invariants and quantum field concepts are merged together (see below). Interesting generalizations to more dimensions, black-hole environments, and gyroscopic precession have been investigated by Iyer and Vishveshwara (1988) (see also Bini *et al.* (1999).

We also notice that at the strictly classical level, the Frenet–Serret coordinate system is quite used in accelerator physics to express Hamiltonians and/or fields (Lee, 2000).

2.1. Stationary World Lines and the Vacuum Excitation of Noninertial Detectors

In 1981, Letaw studied the stationary world lines, on which quantized field detectors in a scalar vacuum have time-independent excitation spectra. They are characterized by the requirement that the geodetic interval between two points depends only on the proper time interval. He used a generalization of the Frenet equations to Minkowski space and found, not surprisingly, that the curvature invariants are the proper acceleration and angular velocity of the world line. Solving the generalized Frenet equations for the simple case of constant invariants leads to several classes of stationary world lines. Letaw gave a classification into six types reviewed here. He also demonstrated the equivalence of the time-like Killing vector field orbits and the stationary world lines. Last but not least, Letaw did some calculations of the vacuum excitation spectra of detectors on the sample of six families of stationary world lines. Letaw's work has a link with Unruh's famous interpretation concerning the excitation of a scalar-particle detector moving with constant linear acceleration in the massless scalar vacuum of flat spacetime that "looks" to such particles as a thermal state of Unruh(!) scalar particles of temperature given by acceleration/2π.

According to DeWitt, the probability for a detector moving along a world line $x^\mu(\tau)$ to be found in an excited state of energy E at $\tau = \tau_0$ is given in terms of the autocorrelation function of the field (Wightman function)

$$P(E) = D(E) \int_{-\infty}^{\tau_0} d\tau \int_{-\infty}^{\tau_0} d\tau' e^{-iE(\tau-\tau')} \langle 0|\phi(x(\tau))\phi(x(\tau'))|0\rangle, \qquad (2)$$

where $D(E)$ is a function characterizing the sensitivity of the detector. The Wightman function for a scalar field is proportional to the inverse of the geodetic interval $\Delta(\tau, \tau')$

$$\langle 0|\phi(x(\tau))\phi(x(\tau'))|0\rangle = [2\pi^2\Delta(\tau, \tau')]^{-1}, \tag{3}$$

where

$$\Delta(\tau, \tau') = [x_\mu(\tau) - x_\mu(\tau')][x^\mu(\tau) - x^\mu(\tau')]. \tag{4}$$

The rate of excitation to the state with energy E is the Fourier cosine transform

$$\frac{dp(E)}{d\tau_0} = 2D(E) \int_{-\infty}^{0} ds \, \langle 0|\phi(x(\tau_0))\phi(x(\tau_0 + s))|0\rangle \cos(Es). \tag{5}$$

where $s = \tau - \tau'$ is the proper time interval. The rate of excitation (the response) is directly related to the energy spectrum of the detected scalar "particles"

$$S(E, \tau) = 2\pi\rho(E) \int_{-\infty}^{0} ds \, \langle 0|\phi(x(\tau))\phi(x(\tau + s))|0\rangle \cos(Es). \tag{6}$$

If the Wightman function is time independent the detected spectra are stationary. This is equivalent to the following property of the geodetic interval

$$\Delta(\tau, \tau + s) = \Delta(0, s). \tag{7}$$

2.1.1. Curvature Invariants and the Frenet Equations

An arbitrary time-like world line in flat space is generally described by four functions, $x^\mu(s)$, specifying the coordinates of each point s on the curve. This parameter may be taken to be the arc length or proper time on the world line. The parametric representation is unsatisfactory in two respects:

(1) A world line is a geometric object and should not require a coordinate-dependent entity for its definition.
(2) There is an inherent redundancy in the parametric representation since three functions suffice to determine the world line.

The Frenet–Serret (curvature) invariants, on the other hand, provide an intrinsic definition of the world line not subject to these criticisms.

The starting point is the construction of an orthonormal tetrad $V_a^\mu(s)$ at every point on the world line $x^\mu(s)$. The Latin index everywhere is a tetrad index. The tetrad is formed from the derivatives of $x^\mu(s)$ with respect to proper time (represented by one or more dots). It is assumed that the first four derivatives are linearly independent, the results being practically unchanged when they are not. Members of the tetrad must satisfy the orthonormality condition

$$V_{a\mu} V_b^\mu = \eta_{ab}, \tag{8}$$

where the metric has diagonal components $(1, -1, -1, -1)$ only.

By Gramm–Schmidt orthogonalization of the derivatives working upwords from the first, the following expressions for the tetrad are found:

$$V_0^\mu = \dot{x}^\mu, \tag{9}$$

$$V_1^\mu = \frac{\ddot{x}^\alpha}{(-\ddot{x}_\alpha \ddot{x}^\alpha)^{1/2}}, \tag{10}$$

$$V_2^\mu = \frac{(\ddot{x}_\gamma \ddot{x}^\gamma)\,\dddot{x}^\mu - (\ddot{x}_\gamma\,\dddot{x}^\gamma)\ddot{x}^\mu + (\ddot{x}_\gamma \ddot{x}^\gamma)^2 \dot{x}^\mu}{[(\ddot{x}_\alpha \ddot{x}^\alpha)^4 + (\ddot{x}_\alpha \ddot{x}^\alpha)(\ddot{x}_\beta\,\dddot{x}^\beta)^2 - (\ddot{x}_\alpha \ddot{x}^\alpha)^2(\dddot{x}^\beta\dddot{x}^\beta)]^{1/2}}, \tag{11}$$

$$V_a^\mu = \frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma\mu} V_{0\alpha} V_{1\beta} V_{2\gamma}, \tag{12}$$

Overall signs on these vectors are fixed by the orientation of the tetrad.

The tetrad V_σ^μ is a basis for the vector space at a point on the world line. Derivatives of the basis vectors may therefore be expanded in terms of them:

$$\dot{V}_a^\mu = K_a^b V_b^\mu. \tag{13}$$

These are the generalized Frenet equations. K_{ab} is a coordinate-independent matrix whose structure must be determined.

Differentiation of the orthonormality condition (8) yields

$$\dot{V}_{a\mu} V_b^\mu + V_{a\mu} \dot{V}_b^\mu = 0, \tag{14}$$

and, in view of (13),

$$K_{ab} = -K_{ba}. \tag{15}$$

A basis vector V_a^μ is defined in terms of the first $a + 1$ derivatives of x^μ; therefore, $\dot{V}_a^\mu$ will be a linear combination of the first $a + 2$ derivatives. These $a + 2$ derivatives are dependent only on the basis vectors V_b^μ where $b \leq a + 1$. This and (15) limit the matrix to the form

$$K_{ab} = \begin{pmatrix} 0 & -\kappa(s) & 0 & 0 \\ \kappa(s) & 0 & -\tau_1(s) & 0 \\ 0 & \tau_1 & 0 & -\tau_2 \\ 0 & 0 & \tau_2 & 0 \end{pmatrix} \tag{16}$$

The three functions of proper time are the invariants

$$\kappa = V_{0\mu} \dot{V}_1^\mu = -\dot{V}_{0\mu} V_1^\mu, \tag{17}$$

$$\tau_1 = V_{1\mu} \dot{V}_2^\mu = -\dot{V}_{1\mu} V_2^\mu, \tag{18}$$

$$\tau_2 = V_{2\mu}\dot{V}_3^{\mu} = -\dot{V}_{2\mu}V_3^{\mu}. \tag{19}$$

They are, respectively, the curvature, the first torsion, and the second torsion (hypertorsion) of the world line. Sign choices are made for reasons brought out below.

To explore the physical significance of the invariants we examine the infinitesimal Lorentz transformations of the tetrad at a point on the world line. The transformations leave the metric invariant

$$\eta_{ab} = L_a^c L_b^d \eta_{cd}. \tag{20}$$

An infinitesimal transformations may be written

$$L_a^c = \delta_a^c + d\epsilon_a^c, \tag{21}$$

where the elements of $d\epsilon_a^c$ are small and must satisfy

$$d\epsilon_{ab} = -d\epsilon_{ba}, \tag{22}$$

The transformations are taken to be active; that is, the transformed tetrad moves by $+v$ and is rotated by θ relative to the untransformed tetrad. Thus the infinitesimal generator is

$$d\epsilon_{ab} = \begin{pmatrix} 0 & -dv_1 & -dv_2 & -dv_3 \\ dv_1 & 0 & -d\theta_{12} & d\theta_{13} \\ dv_2 & d\theta_{12} & 0 & -d\theta_{23} \\ dv_3 & -d\theta_{31} & d\theta_{23} & 0 \end{pmatrix} \tag{23}$$

The change in the tetrad resulting from this transformation is

$$\dot{V}_a^{\mu} = \left(d\epsilon_a^b/ds\right)V_b^{\mu}. \tag{24}$$

Equations (24) are identical to the Frenet Eqs. (13); therefore, the physical content of the curvature invariants is found by comparison of (16) and (23).

2.1.2. Physical Interpretation of the FS Invariants

The following interpretations of the FS kinematic invariants is quite well known

κ is the proper acceleration of the world line which is always parallel to V_1^{ν}. τ_1 and τ_2 are the components of proper angular velocity of the world line in the planes spanned by V_1^{μ} and V_2^{μ}, and V_2^{μ} and V_3^{μ}, respectively. The total proper angular velocity is the vector sum of these two invariants.

2.1.3. Stationary Motions

The simplest world lines are those whose curvature invariants are constant. They are called stationary because their geometric properties are independent of proper time. One also finds that only observers on these world lines may establish a coordinate system in which they are at rest and the metric is stationary. Clearly, the geodetic interval between two points on a stationary world line can depend only on the proper time interval, therefore they are the world lines on which a detector's excitation is time independent.

The Frenet Eq. (13) may be reduced to a fourth-order linear Equation in V_0^ν when the curvature invariants are constant

$$\dddot{V}_0^\mu - 2a\ddot{V}_0^\mu - b^2 V_0^\mu = 0, \tag{25}$$

where

$$a = \tfrac{1}{2}(\kappa^2 - \tau_1^2 - \tau_2^2), \qquad b = |\kappa\tau_2|.$$

The other basis vectors are determined from V_0^μ by the Equations

$$V_1^\mu = \dot{V}^\mu/\kappa \tag{26}$$

$$V_2^\mu = (\ddot{V}_0^\mu - \kappa^2 V_0^\mu)/\kappa\tau_1 \tag{27}$$

$$V_3^\mu = \left[\dddot{V}_0^\mu - (\kappa^2 - \tau_1^2)\dot{V}_0^\mu\right]/\kappa\tau_1\tau_2. \tag{28}$$

Equation (25) is homogeneous with constant coefficients. The four roots of the characteristic equation are $\pm R_+$ and $\pm i R_-$, where

$$R_\pm = \left[(a^2 + b^2)^{1/2} \pm a\right]^{1/2}. \tag{29}$$

The solution is of the form

$$V_0^\mu = A^\mu \cosh(R_+ s) + B^\mu \sinh(R_+ s) C^\mu \cos(R_- s) + D^\mu \sin(R_- s). \tag{30}$$

Using (26)–(28) and (30) at $s = 0$ and the initial conditions $(V_a^\mu)_{s=0} = \delta_a^\mu$ one can get the following expressions for the coefficients

$$A^\mu = R^{-2}(R_-^2 + \kappa^2, 0, \kappa\tau_1, 0), \tag{31}$$

$$B^\mu = R^{-2}(0, \kappa(R_-^2 + \kappa^2 - \tau^2)/R_+, 0, \kappa\tau_1\tau_2/R_+), \tag{32}$$

$$C^\mu = R^{-2}(R_+^2 - \kappa^2, 0, -\kappa\tau_1, 0), \tag{33}$$

$$D^\mu = R^{-2}(0, \kappa(R_+^2 - \kappa^2 - \tau^2)/R_-, 0, -\kappa\tau_1\tau_2/R_-), \tag{34}$$

with $R^2 = R_+^2 + R_-^2$. From these results on the FS tetrad one gets easily the six classes of stationary world lines and the corresponding excitation spectra.

2.1.4. The Six Stationary Scalar Frenet–Serret Radiation Spectra

1. $\kappa = \tau_1 = \tau_2 = 0$ (inertial, uncurved world lines)
   The excitation spectrum is a trivial cubic spectrum

$$S_{\text{recta}}(E) = \frac{E^3}{4\pi^2}, \tag{35}$$

   i.e., as given by a vacuum of zero point energy per mode $E/2$ and density of states $E^2/2\pi^2$.

2. $\kappa \neq 0, \tau_1 = \tau_2 = 0$ (hyperbolic world lines)
   The excitation spectrum is Planckian allowing the interpretation of $\kappa/2\pi$ as "thermodynamic" temperature. In the dimensionless variable $\epsilon_\kappa = E/\kappa$ the vacuum spectrum reads

$$S_{\text{hyp}}(\epsilon_\kappa) = \frac{\epsilon_\kappa^3}{2\pi^2(e^{2\pi\epsilon_\kappa-1})}. \tag{36}$$

3. $\kappa| < |\tau_1|, \tau_2 = 0, \rho^2 = \tau_1^2 - \kappa^2$ (helical world lines)
   The excitation spectrum is an analytic function corresponding to the case 4 below only in the limit $\kappa \gg \rho$

$$S_{\text{hel}}(\epsilon_\rho) \overset{\kappa/\rho\to\infty}{\longrightarrow} S_{3/2-\text{parab}}(\epsilon_\kappa). \tag{37}$$

   Letaw plotted the numerical integral $S_{\text{hei}}(\epsilon_\rho)$, where $\epsilon_\rho = E/\rho$ for various values of κ/ρ.

4. $\kappa = \tau_1, \tau_2 = 0,$ (if spatially projected: semicubical parabolas $y = \frac{\sqrt{2}}{3}\kappa x^{3/2}$)
   The excitation spectrum is analytic, and since there are two equal curvature invariants one can use the dimensionless energy variable ϵ_κ

$$S_{3/2-\text{parab}}(\epsilon_\kappa) = \frac{\epsilon_\kappa^2}{8\pi^2\sqrt{3}}e^{-2\sqrt{3}\epsilon_\kappa}. \tag{38}$$

   It is worth noting that $S_{3/2-\text{parab}}$, being a monomial times an exponential, is quite close to the Wien-type spectrum $S_W \propto \epsilon^3 e^{-\text{const}.\epsilon}$.

5. $|\kappa| > |\tau_1|, \tau_2 = 0, \sigma^2 = \kappa^2 - \tau_1^2$ (if spatially projected: catenaries $x = \kappa \cosh(y/\tau)$)
   In general, the catenary spectrum cannot be found analytically. It is an intermediate case, which for $\tau/\sigma \to 0$ tends to S_{hyp}, whereas for $\tau/\sigma \to \infty$ tends toward $S_{3/2-\text{parab}}$

$$S_{\text{hyp}}(\epsilon_\kappa) \overset{0\leftarrow\tau/\sigma}{\longleftarrow} S_{\text{catenary}}(\epsilon_\sigma) \overset{\tau/\sigma\to\infty}{\longrightarrow} S_{3/2-\text{parab}}(\epsilon_\kappa). \tag{39}$$

6. $\tau_2 \neq 0$ (rotating world lines uniformly accelerated normal to their plane of rotation)

The excitation spectrum is given in this case by a two-parameter set of curves. These trajectories are a superposition of the constant linearly accelerated motion and uniform circular motion. The corresponding vacuum spectra have not been calculated by Betaw, not even numerically.

Thus, only the hyperbolic world lines, having just one nonzero curvature invariant, allow for a Planckian excitation spetrum and lead to a strictly one-to-one mapping between the curvature invariant κ and the "thermodynamic" temperature $(T_{\text{hyp}} = \kappa/2\pi)$ The excitation spectrum due to semicubical parabolas can be fitted by Wien-type spectra, the radiometric parameter corresponding to both curvature and torsion. The other stationary cases, being nonanalytical, lead to approximate determination of the curvature invariants, denning locally the classical world line on which a relativistic quantum particle moves.

To this end, I mention that Letaw introduced the terminology *ultratorsional, paratorsional, infratorsional*, and *hypertorsional* for the scalar stationary cases 3–6, respectively.

2.2. The Electromagnetic Vacuum Radiation

For the case of homogeneous electromagnetic field, Honig *et al.* proved that the FS scalars remain constant along the world line of a charged particle, whereas the FS vectors obey a Lorentz force equation of the form

$$\dot{u}^{\mu} = \bar{F}^{\mu}_{\nu} u^{\nu} \tag{40}$$

with $F_{\mu\nu} = \lambda \bar{F}^{\mu}_{\nu}$, the electromagnetic field tensor, $\lambda = e/mc^2$ and $\dot{u}^{\mu}$ the four-velocity of the particle.

A remarkable physical interpretation of the FS invariants in terms of the Lorentz invariants of the electromagnetic field was established by Honig *et al.*

$$\kappa^2 - \tau_1^2 - \tau_2^2 = \lambda^2(E^2 - H^2), \qquad \kappa\tau_2 = \lambda^2(\vec{E} \cdot \vec{H}). \tag{41}$$

These beautiful results passed quite unnoticed until now. They could be used for a geometric transcription of homogeneous electromagnetism and therefore for the geometric "calibration" of electromagnetic phenomena.

The FS formalism has not been used for the electromagnetic vacuum noise. Other approaches have been undertaken for this important case of which a rather complete one is due to Hacyan and Sarmiento that is briefly presented in the following.

2.2.1. The Hacyan–Sarmiento Approach

Starting with the expression for the electromagnetic energy–momentum tensor

$$T_{\mu\nu} = \frac{1}{16\pi} \left(4F_{\mu\alpha} F_{\nu}^{\alpha} + \eta_{\mu\nu} F_{\lambda\beta} F^{\lambda\beta} \right), \tag{42}$$

Hacyan–Sarmiento define the electromagnetic two-point Wightman functions as follows

$$D_{\mu\nu}^{+}(x, x') \equiv \tfrac{1}{4} \left(4F_{(\mu}^{\alpha}(x) F_{\nu)\alpha}(x') + \eta_{\mu\nu} F_{\lambda\beta}(x) F^{\lambda\beta}(x') \right); \tag{43}$$

$$D_{\mu\nu}^{-}(x, x') \equiv D_{\mu\nu}^{+}(x', x). \tag{44}$$

This may be viewed as a variant of the "point-splitting" approach advocated by DeWitt. Moreover, because of the following mathematical properties

$$\eta^{\mu\nu} D_{\mu\nu}^{\pm} = 0, \qquad D_{\mu\nu}^{\pm} = D_{\nu\mu}^{\pm}, \qquad \partial_{\nu} D_{\mu}^{\pm\nu} = 0, \tag{45}$$

the electromagnetic Wightman functions can be expressed in terms of the scalar Wightman functions as follows

$$D_{\mu\nu}^{\pm}(x, x') = c\partial_{\mu} \partial_{\nu} D^{\pm}(x, x'), \tag{46}$$

where c is in general a real constant depending on the case under study. This shows that from the standpoint of their vacuum fluctuations the scalar and the electromagnetic fields are not so different.

Now introduce sum and difference variables

$$s = \frac{t + t'}{2}; \qquad \alpha = \frac{t - t'}{2}. \tag{47}$$

Using the Fourier transforms of the Wightman functions

$$\tilde{D}^{\pm}(\omega, s) = \int_{-\infty}^{\infty} d\sigma e^{-iw\sigma} D^{\pm}(s, \sigma), \tag{48}$$

where ω is the frequency of zero-point fields, the *particle number density of the vacuum seen by the moving detector and the spectral vacuum energy density per mode* are given by

$$n(\omega, s) = \frac{1}{(2\pi)^2\omega} \left[\tilde{D}^{+}(\omega, s) - \tilde{D}^{-}(\omega, s) \right], \tag{49}$$

$$\frac{de}{d\omega} = \frac{\omega^2}{\pi} \left[\tilde{D}^{+}(\omega, s) + \tilde{D}^{-}(\omega, s) \right]. \tag{50}$$

Circular World Lines. The most important application of these results is to a uniformly rotating detector whose proper time is s and angular speed is ω_0 in motion along the circular world line

$$x^\alpha(s) = (\gamma s, R_0 \cos(\omega_0 s), R_0 \sin(\omega_0 s), 0), \qquad (51)$$

where R_0 is the rotation radius in the inertial frame, $\gamma = (1 - v^2)^{-1/2}$, and $v = \omega_0 R_0 / \gamma$. In this case there are two Killing vectors $k^\alpha = (1, 0, 0, 0)$ and $m^\alpha(s) = (0, -R_0 \sin(\omega_0 s), R_0 \cos(\omega_0 s), 0)$. Expressing the Wightman functions in terms of these two Killing vectors, HS calculated the following physically observable spectral quantities (i.e., those obtained after subtracting the inertial zero-point field contributions):

- The spectral energy density

$$\frac{de}{d\omega} = \frac{\gamma^3}{2\pi^3 R_0^3} \frac{\omega^2 + (\gamma v \omega_0)^2}{\omega^2} \frac{v^3 w^2}{w^2 + (2\gamma v)^2} h_\gamma(w), \qquad (52)$$

- The spectral flux density

$$\frac{dp}{d\omega} = \frac{\gamma^3}{2\pi^3 R_0^3} \frac{\omega^2 + (\gamma v \omega_0)^2}{\omega^2} 4v^4 k_\gamma(w), \qquad (53)$$

- The spectral stress density

$$\frac{ds}{d\omega} = \frac{\gamma^3}{2\pi^3 R_0^3} \frac{\omega^2 + (\gamma v \omega_0)^2}{\omega^2} \frac{v^3 w^2}{w^2 + (2\gamma v)^2} j_\gamma(w). \qquad (54)$$

The ratio $(\omega^2 + (\gamma v \omega_0)^2)/\omega^2$ is a density-of-states factor introduced for convenience.

The Hacyan–Sarmiento variables are $w = \frac{2\omega}{\omega_0}$ and $x = \frac{\sigma \omega_0}{2}$, whereas $h_\gamma(w)$, $k_\gamma(w)$ and $j_\gamma(w)$ are the following cosine-Fourier transforms

$$h_\gamma(w) \equiv \int_0^\infty \left(\frac{N_h(x, v)}{\gamma^2 [x^2 - v^2 \sin^2 x]^3} - \frac{3}{x^4} + \frac{2\gamma^2 v^2}{x^2} \right) \cos(wx)\, dx; \qquad (55)$$

$$k_\gamma(w) \equiv -\int_0^\infty \left(\frac{N_k(x, v)}{\gamma^2 [x^2 - v^2 \sin^2 x]^3} - \frac{3}{x^4} - \frac{\gamma^2}{6x^2} \right) \cos(wx)\, dx; \qquad (56)$$

$$j_\gamma(w) \equiv \int_0^\infty \left(\frac{1}{\gamma^4 [x^2 - v^2 \sin^2 x]^2} - \frac{1}{x^4} + \frac{2\gamma^2 v^2}{3x^2} \right) \cos(wx)\, dx. \qquad (57)$$

The numerators $N_h(x, v)$ and $N_k(x, v)$ are given by

$$N_h(x, v) = (3 + v^2)x^2 + (v^2 + 3v^4) \sin^2 x - 8v^2 \sin x; \qquad (58)$$

$$N_k(x, v) = x^2 + v^2 \sin^2 x - (1 + v^2)x \sin x. \qquad (59)$$

Ultra-relativistic Limit: $\gamma \gg 1$. The functions $H_\gamma = \frac{v^3 w^2}{w^2 + (2\gamma v)^2} h_\gamma(w)$, $K_\gamma = 4v^4 k_\gamma(w)$, $J_\gamma = \frac{v^3 w^2}{w^2 + (2\gamma v)^2} j_\gamma(w)$, all have the following scaling property $X_{k\gamma}(kw) = k^3 X_\gamma(w)$ where k is an arbitrary constant, and $X = H, K, J$, respectively. This is the same scaling property as that of a Planckian distribution with a temperature proportional to γ.

Nonrelativistic Limit: $\gamma \ll 1$. A detailed discussion of the nonrelativistic-limit for the scalar case has been provided by Kim *et al.* (1987), who used the parameters v and ω_0, and not acceleration and speed as used by Letaw and Pfautsch for the circular scalar case (Letaw and Pfautsch, 1980). They obtained a series expansion in velocity

$$\frac{de}{d\omega} = \frac{\omega^3}{\pi^2} \left[\frac{\omega_0}{\gamma \omega} \sum_{n=0}^{\infty} \frac{v^{2n}}{2n+1} \sum_{k=0}^{n} (-1)^k \frac{(n - k - \frac{\omega}{\gamma \omega_0})^{2n+1}}{k!(2n-k)!} \Theta \left(n - k - \frac{\omega}{\gamma \omega_0} \right) \right],$$
(60)

where Θ is the usual Heavy-side step function. Thus, to a specified power of the velocity many vacuum harmonics could contribute; making the energy density spectrum quasi-continuous. For low frequencies the difference between the scalar and electromagnetic case is small. Besides, one can consider only the first few terms in the series as an already good approximation.

2.3. Nonstationary Vacuum Radiation Patterns

Nonstationary vacuum field radiation has a time-dependent spectral content requiring joint time and frequency information, i.e., one needs generalizations of the power spectrum concept. One can think of (i) tomographical processing and/or (ii) wavelet transforms. For instance, the recently proposed noncommutative to-mography (NCT) transform $M(s; \mu, \nu)$ (Man'ko and Vilela Mendes, 1999), if relativistically generalized, could be an attractive way of processing nonstationary signals. In the definition of M, s is just an arbitrary curve in the noncommutative time-frequency plane, while μ, and ν are parameters characterizing the curve. The most simple examples are the axes $s = \mu t + \nu \omega$, where μ and ν are linear combination parameters. The noncommutative tomography transform is related to the Wigner–Ville quasi-distribution $W(t, \omega)$ by an invertible transformation and has the following useful properties

$$M(t; 1, 0) = |f(t)|^2,$$
(61)

$$M(\omega; 0, 1) = |f(\omega)|^2,$$
(62)

where f is the analytic signal which is simulated by M. Furthermore, employing M leads to an enhanced detection of the presence of signals in noise which has a small signal-to-noise ratio. The latter property may be very useful in detecting

vacuum field radiation noises, which are very small "signals" with respect to more common noise sources.

On the other hand, since in the quantum detector method, the vacuum auto-correlation functions are the essential physical quantities. In addition, according to various fluctuation–dissipation theorems these functions are related to the linear (equilibrium) response functions to an initial condition/vacuum. The fluctuation–dissipation approach has been developed and promoted by Sciama and collaborators (Candelas and Sciama, 1977). In principle, the generalization of the fluctuation–dissipation theorem for some classes of out of equilibrium relaxational systems, such as glasses, looks also promising for the case of nonstationary vacuum noise. One can use a so-called two-time fluctuation–dissipation ratio $X(t, t')$ and write a modified fluctuation–dissipation relationship (Cugliandolo *et al.*, 1997),

$$T_{\text{eff}}(t, t')R(t, t') = X(t, t')\frac{\partial \tilde{C}(t, t')}{\partial t'} \tag{63}$$

where R is the response function and $\tilde{C}$ the autocorrelation function. The fluctuation–dissipation ratio is employed to perform the separation of scales. Moreover, T_{eff} are timescale-dependent quantities, making them promising for relativistic vacuum radiation, which corresponds naturally to out of equilibrium conditions.

The necessity of a generalized nonequilibrium formalism, has been also emphasized in the same context but using the influence functional formalism (Feynman and Vernon, 1963), by Hu and Johnson (2002). The basic idea is to treat the quantum field vacuum as a sort of coarse-grained environment through which the quantum particles move.

3. DETECTION PROPOSALS

A number of model experiments to detect the Frenet–Serret vacuum radiation patterns have been suggested in the last 20 years (Rosu, 2001, 2002). Because the curvature thermodynamic temperature is given by $T_\kappa = \frac{\hbar}{2\pi ck}a$ a this leads to $T_\kappa = 4.10^{-23} a$ and one needs accelerations greater than $10^{20} g_\oplus$ to have "thermal" effects of only a few Kelvins. On the other hand, one should focus below the Schwinger acceleration for copious spontaneous pair creation out of QED vacuum

$$a_{\text{Schw}} \approx m_e c^3/\hbar \approx 10^{29} m/s^2 \approx 10^{28} g_\oplus, \tag{64}$$

There are indeed several physical settings in which accelerations can be achieved only a few orders below the Schwinger acceleration and forthcoming technological advances could test routinely those acceleration scales. The stationary FS vacuum radiation patterns could be revealed as tiny signals in the background of by far more powerful effects.

3.1. Vacuum Radiation Effects in Storage Rings ($a \sim 10^{22} g_\oplus$, $T_\kappa = 1200$ K)

Bell and Leinaas imagined the first laboratory phenomenon connected to the FS radiation effects. During 1983–1987 they published a number of papers on the idea that the depolarising effects in electron storage rings could be interpreted in terms of FS radiation effects (Bell and Leinaas, 1983; Bell and Hughes, 1985; Bell and Leinaas, 1987; McDonald, 1987) However, the incomplete radiative polarization of the electrons in storage rings has been first predicted in early sixties in the framework of QED by Sokolov and Ternov. Besides, the circular vacuum noise is not sufficiently "universal" since it always depends on both acceleration and velocity. This appears as a drawback of the "storage ring electron thermometry," not to mention the very intricate spin physics. Keeping these facts in mind, we go on with further comments, following the nontechnical discussion of Leinaas (1991).

The circular acceleration in the LEP machine is $a_{\mathrm{LEP}} = 3 \times 10^{22} g_\oplus$ corresponding to the kappa temperature $T_\kappa = 1200°K$. It is a simple matter to show that an ensemble of electrons in a uniform magnetic field at a nonzero temperature will have a polarization expressed through the following hyperbolic tangent $P_\kappa = \tanh(\frac{\pi g}{2\beta})$. For the classical value of the gyromagnetic factor ($g = 2$) and for highly relativistic electrons ($\beta = 1$), $P_\kappa = \tanh \pi = 0.996$, beyond the limiting polarization of Sokolov and Ternov $P_{\mathrm{ST,max}} = \frac{8\sqrt{3}}{15} = 0.924$ (Sokolov and Ternov, 1963).

On the other hand, the function

$$P_\kappa(g) = \tanh\left(\frac{\pi g}{2\beta}\right) = \frac{1 - e^{-\pi g}}{1 + e^{-\pi g}} \tag{65}$$

is very similar, when plotted, to the function $P_{\mathrm{DK}}(g)$, which is a combination of exponential and polynomial terms in the anomalous part of the gyromagnetic factor of the electron, and it was obtained through QED calculations by Derbenev and Kondratenko (1973),

$$P_{\mathrm{eq}}^{\mathrm{DK}} = \frac{8}{5\sqrt{3}} \frac{\langle |\rho|^{-3} \hat{b}(\hat{n} - F_{\mathrm{DK}}) \rangle}{\langle |\rho|^{-3}(1 - 2/9(\hat{n} \cdot \hat{v})^2 + 11/18|F_{\mathrm{DK}}|^2) \rangle}, \tag{66}$$

where $F_{\mathrm{DK}} = \gamma \frac{\partial \hat{n}}{\partial \gamma}$ is the spin–orbit coupling function, which takes into account the depolarizing effects of jumps between various trajectories differing from the reference closed orbit, ρ is the bending radius, $\hat{b}$ a unit vector along the transverse magnetic field component. The brackets indicate an average over the ring circumference and over the ensemble of particles in the beam. The unit vector $\hat{n}$ is the time-independent spin solution of the BMT equation, attached to each particle trajectory.

The difference between P_κ and P_{DK} is merely a shift of the latter along the positive g-axis with about 1.2 units. As shown by Bell and Leinaas, when

the Thomas precession of the electron is included in the spin Hamiltonian, a shift of 2 units is obtained for $P_\kappa(g)$. This suggests a more careful treatment of spin effects arising when one is going from the lab system to the circulating coordinate frame. A new spin Hamiltonian was introduced by Bell and Leinaas with a more complicated structure of the field vector in the scalar product with the Pauli matrices. This complicated structure takes into account the classical external fields, the quantum radiation field and the fluctuations around the classical path. Within this more complete treatment, Bell and Leinaas were able to get, to linear order in the quantum fluctuations, a Thomas-like term and a third resonant term. The latter is directly related to the vertical fluctuations in the electron orbit, which are responsible for the spin transitions

$$P_{\text{BL}} = \frac{8}{5\sqrt{3}} \frac{1 - \frac{f}{6}}{1 - \frac{f}{18} + \frac{13}{360} f^2} \tag{67}$$

where the resonance factor f reads

$$f = \frac{2a_g Q^2}{Q_z^2 - a_g^2 \gamma^2}. \tag{68}$$

Q_z is the vertical betatron tune, γ is the Lorentz factor and $a_g = (g-2)/2$ is the electron g anomaly. This induces an interesting variation of the beam polarization close to the resonance. As γ passes through it from below, the polarization first falls from 92 to -17%, and then it increases again to 99% before stabilizing to 92%. This is the only clear difference from the standard QED. Such resonances induced by the vertical fluctuations of the orbit have been considered before in the literature within classical spin diffusion models and focused strictly on their depolarizing effect. Their nature is related to the fact that the Fourier spectrum of the energy jumps associated with the quantum emission processes contains harmonics giving the usual resonance condition. As emphasized by Bell and Leinaas, a more direct experimental demonstration of the circular vacuum noise would be the measurement of the vertical fluctuations. However, this will clearly be a very difficult task since such fluctuations are among the smallest orbit perturbations. At the same time, the measurement of the polarization variation close to the narrow resonance, in particular the detection of transient polarizations exceeding the Sokolov–Ternov limiting one, will make us more confident in the claims of Bell and Leinaas. It is worth mentioning that the rapid passage through the resonance does not change the polarization, while a slow passage reverses it but does not change the degree of polarization. Therefore, only an intermediate rate with respect to the quantum emission time scale of passing through the resonance will be appropriate for checking the transient BL effects. Barber and Mane (1988), have shown that the DK and BL formalisms for the equilibrium degree of radiative electron polarization are not so different as they might look. They also obtained

an even more general formula for P_{eq} than the DK and BL ones and from their formula they estimated as negligible the BL increase near the resonance. It reduces to the following substitution of the resonance factor

$$f \rightarrow f + \frac{2}{\gamma}. \tag{69}$$

The basic experimental data on spin depolarizing effects remain as yet those measured at SPEAR at energies around 3.6 GeV in 1983. Away from the resonant γ's the maximum polarization of Sokolov and Ternov was confirmed (Johnson *et al.*, 1983), but no dedicated study of the polarization rates across resonances have been performed.

A paper of Cai *et al.* (1993), claims that the Mashhoon effect due to the spin-rotation coupling is stronger than the circular vacuum effect (spin-acceleration coupling) at all accelerator energies in the case of a perfect circular storage ring. However, the comparison is not at all a straightforward one.

Bautista (1993), solved the Dirac equation in Rindler coordinates with a constant magnetic field in the direction of acceleration and showed that the Bogolubov coefficients of this problem do not mix up the spin components. Thus there is no spin polarization due to the acceleration in this case.

In our opinion, the real importance of considering vacuum radiation effect at storage rings is related to clarifying radiometric features of the synchrotron radiation (Rosu, 1994). There is a strong need to establish radiometric standards in spectral ranges much beyond those of the cavity/blackbody standards, and synchrotron radiation has already been considered experimentally from this point of view (Kühne, 1989). Quantum field thermality is intrinsically connected to the KMS condition. This is a well-known skew periodicity in imaginary time of Green's functions expressing the detailed balance criterion in field theory. However, the task is to work out in more definite terms the radiometric message of the KMS quantum/stochastic processes (Klein and Landau, 1981).

Recently, the spin-flip synchrotron radiation has been experimentally shown to be important in the hard part of the spectrum in the axial channeling of electrons in the energy range 35–243 GeV incident on a W single crystal (Kirsebom *et al.*, 2001; Korol *et al.*, 2002). This may revive the interest in the BL approach, especially in the cleaner planar channeling case (Uggerhøj, private communication).

3.2. Vacuum FS Radiation and Geonium Physics ($a \sim 10^{21} g_\oplus$, $T_\kappa = 2.4$ K)

The very successful Geonium physics could help detecting the circular thermal-like vacuum noise. The proposal belongs to Rogers (1988), being one of the most attractive. The idea of Rogers is to place a small superconducting Penning

trap in a microwave cavity. A single electron is constrained to move in a cyclotron orbit around the trap axis by a uniform magnetic field (Rogers' figure is $B = 150$ kGs). The circular proper acceleration is $a = 6 \times 10^{21} g_\oplus$ corresponding to $T_\kappa = 2.4$ K. The velocity of the electron is maintained fixed ($\beta = 0.6$) by means of a circularly polarized wave at the electron cyclotron frequency, compensating also for the irradiated power. The static quadrupole electric field of the trap creates a quadratic potential well along the trap axis in which the electron oscillates. The axial frequency is 10.5 GHz (more than 150 times the typical experimental situation (Brown and Gabrielse, 1986)) for the device scale chosen by Rogers. This is the measured frequency because it is known (Brown and Gabrielse, 1986), that the best way of observing the electron motion from the outside world (Feynman's "rest of the Universe") is through the measurement of the current due to the induced charge on the cap electrodes of the trap, as a consequence of the axial motion of the electron along the symmetry axis. At 10.5 GHz the difference in energy densities between the circular FS radiation and the universal linear FS radiation are negligible (see Fig. 2 in Rogers' work). Actually, Rogers used the parametrization for the spectral energy density of a massless scalar field as given by Kim *et al.* (1987), that he wrote in the form

$$\frac{de}{d\omega} = \frac{\hbar}{\pi^2 c^3} \left[\frac{\omega^3}{2} + \gamma \omega_c^3 x^2 \sum_{n=0}^{\infty} \frac{\beta^{2n}}{2n+1} \sum_{k=0}^{n} (-1)^k \frac{n-k-x}{k!(2n-k)} \Theta[n-k-x] \right] \tag{70}$$

where γ is the relativistic gamma factor, $x = \omega/\gamma\omega_c$, and $\omega_c = eB/\gamma mc$ is the cyclotron frequency. The power spectral density at the axial frequency is only $\partial P/\partial f = 0.47 \times 10^{-22}$ W/Hz, and may be assumed to be almost the same as the electromagnetic spectral energy density. This power is resonantly transferred to the TM_{010} mode of the microwave cavity and a most sensible cryogenic GaAs field–effect transistor amplifier should be used to have an acceptable signal-to-noise ratio of $S/N = 0.3$. According to Rogers, the signal can be distinguished from the amplifier noise in about 12 ms.

In conclusion, very stringent conditions are required in the model experiment of Rogers. Top electronics and cryogenic techniques are involved as well as the most sophisticated geonium methods. Taking into account the high degree of precision attained by geonium techniques, one may think of Rogers' proposal as one of the most feasible. The critique of this proposal is similar to that in storage rings (Rosu, 1996): the circular FS radiation is not universal, depending also on the electron velocity! In addition, Levin *et al.* (1993), studied the vacuum radiation effects for a massless scalar field enclosed in a two-dimensional circular cavity concluding that the effects of finite cavity size on the frequencies of normal modes of the cavity (Casimir effect) ignored by Bell *et al.* and by Rogers are in fact quite important.

3.2.1. Cylindrical Penning Traps

A better experimental setting for detecting vacuum radiation noises by means of a trapped quantum detector (electron) may well be the cylindrical Penning trap, for which the trap itself is a microwave cavity (Tan and Gabrielse, 1989, 1993). In this case small slits incorporating choke flanges divide high-conductivity copper cavity walls into the required electrode Penning configuration, including two compensation electrodes. The driven axial resonance for this configuration has already been observed with almost the same signal-to-noise ratio as in hyperbolic Penning traps. By means of these cylindrical cavity traps, a more direct coupling to the cavity modes may be achieved, especially in the weak coupling regime. Here the cyclotron oscillator and the cavity mode cannot form normal modes, and therefore other nonlinear effects are not coming into play. The cylindrical TM_{010} mode is essentially a zero-order Bessel function in the radial direction and has no modes along the z axis. The price to pay in the case of the cylindrical trap is a loss of control on the quality of the electrostatic quadrupole potential.

3.3. Vacuum Radiation From Plasma Fronts ($a \sim 10^{20} g_{\oplus}$, $T - \kappa \sim 1\,\mathrm{K}$)

An experimental equivalent of a *fast moving mirror might* be a *plasma front* created when a gas is suddenly photoionized. This is the proposal of Yablonovitch (1989). The argument is that the phase shift of the zero-point electromagnetic field transmitted through a plasma window whose index of refraction is falling with time (from 1 to 0) is the same as when reflected from an accelerating mirror. Consider the case of hyperbolic motion. Since the velocity is

$$v = c \tanh(a\tau/c) \tag{71}$$

where τ is the observer's proper time, the Doppler shift frequency will be

$$\omega_D = \omega_0 \sqrt{\frac{1 - v/c}{1 + v/c}} = \omega_0 \exp(-a\tau/c) \tag{72}$$

and consequently a plane wave of frequency ω_0 turns into a wave with a time-dependent frequency. Such waves are called chirped waves in nonlinear optics and acoustics. Equation (72) represents an *exponential chirping* valid also for black holes. For an elementary discussion of Doppler shift for accelerated motion the reader is directed to a paper of Cochran (1989). It is worthwhile to mention that in the semiclassical treatment of black hole physics one is usually dealing with chirped signals, since the WKB functions are generally of variable wavelength, and by meeting supplementary conditions on their derivatives they are made to look as much as possible like fixed linear combinations of plane waves. On the other hand, in the case of wave packets one is always working with the average

frequency of the wave packets (see, e.g., the paper of Frolov and Novikov on the dynamical origin of black hole entropy (Frolov and Novikov, 1993).)

The technique of producing plasma fronts/windows in a gas by laser break-down, and the associated frequency upshifting phenomena (there are also down-shifts) of the electromagnetic waves interacting with such windows, are well settled since about 20 years. Blue shifts of about 10% have been usually observed in the transmitted laser photon energy.

In his paper, Yablonovitch works out a very simple model of a linear chirping due to a refractive index linearly decreasing with time, $n(t) = n_0 - \dot{n}t$, implying a Doppler shift of the form $\omega \rightarrow \omega[1 + \frac{\dot{n}}{n}t] \sim \omega[1 + \frac{a}{c}t]$. To have accelerations $a = 10^{20} g_\oplus$ the laser pulses should be less than 1 ps. Even more promising may be the nonadiabatic photoionization of a semiconductor crystal in which case the refractive index can be reduced from 3.5 to 0 on the timescale of the optical pulse. As discussed by Yablonovitch, the pump laser has to be tuned just below the Urbach tail of a direct-gap semiconductor in order to create weakly bound virtual electron-hole pairs. These pairs contribute a large reactive component to the photocurrent since they are readily polarized. The background is due to the bremsstrahlung emission produced by real electron-hole pairs, and to diminish it one needs a crystal with a big Urbach slope (the Urbach tail is an exponential behavior of the absorption coefficient).

3.3.1. Ambiguity of the Interpretation

Yablonovitch remarked that the experimental interpretation is highly ambiguous. There are two types of reasoning:

(i) technologically oriented interpretations
1. A single-cycle microwave squeezing.
2. An inverse quadratic electro-optic effect with zero-point photons as input waves.
(ii) concept-oriented interpretations
3. Nonadiabatic Casimir effect.
4. Unruh effect.

In general, one should notice the difference between the laboratory and the black hole/hyperbolic chirping. The former is *linear*, whereas the latter is *exponential*.

The "plasma window" of Yablonovitch was criticized in the important paper by Dodonov, Klimov, and Nikonov [DKN] (Dodonov *et al*., 1993), on the grounds that we are not in the case of exponentially small reflection coefficient as required to get a Planck spectrum from vacuum fluctuations. At the general level, one may argue that nonstationary Casimir effects may produce some deformed Planck distributions, and only in particular cases purely Planck distributions. As a matter of fact, depending on the nonstationarity, one may obtain very peculiar photon

spectra, and this might be of great interest in applied physics. DKN showed explicitly that an exponential "plasma window," for which presumably the modulation "depth" is the effective kappa temperature, does not produce a Planck spectrum. However, for a parametric function displaying the symmetric Epstein profile one can get in the adiabatic limit a "Wien's spectrum" with the effective temperature given by the logarithmic derivative of the variable magnetic permeability with respect to time. According to DKN this corresponds to a "dielectric window" and not to a "plasma window." The experimental realization of nonstationary Casimir effects are either resonators with moving walls, as first discussed by Moore (1970), or resonators with time-dependent refractive media as discussed by DKN. On the lines of Yablonovitch, Hizhnyakov (1992), studied the sudden changes of the refractive index caused by the excitations of a semiconductor near a band-to-band transition in the infrared by a synchronously pumped subpicosecond dye laser, and also referred to the analogy with thermal-like radiation. In 1994, Law (1994), combined the moving walls of Moore with the dielectric medium with time-varying permittivity in a one-dimensional electromagnetic resonant cavity. In this way, he obtained an effective quadratic Hamiltonian, which is always required when we want to discuss nonstationary "particle production" effects.

3.4. Vacuum Radiation and Sonoluminescence

A few years ago, Eberlein (1996), claimed that sonoluminescence could be best interpreted if one goes beyond the perfect mirror restriction. In particular, whenever *an interface between two dielectrics or a dielectric and the vacuum moves noninertially photons are created* (from the vacuum).

Choosing as model profile for the time-dependence of the bubble radius about the collapse

$$R^2(t) = R_0^2 - b\left(R_0^2 - R_{\min}^2\right)\frac{1}{(t/\gamma)^2 + 1}, \tag{73}$$

Eberlein obtained for bubble radius R much greater than the wavelengths of the light emitted the thermal-like spectral density

$$S(\omega) = \frac{(n^2 - 1)^2}{64n^2}\frac{\hbar}{c^4\gamma}\left(R_0^2 - R_{\min}^2\right)^2\omega^3 e^{-2\gamma\omega}, \tag{74}$$

using only zero-temperature quantum field calculations. She also calculated the total energy radiated during one acoustic cycle obtaining $\sim 2 \times 10^{-13}$ in a lapse of 1 fs and claimed that this roughly corresponds to the observed number of photons. Her results were strongly contended by many comments of other people. In particular, Milton (quant-ph/9608003), used the Larmor formula $P = \frac{2}{3}\frac{(\ddot{d})^2}{c^3}$ with the electric dipole moment $d \sim eR$ to estimate the classical Larmor energy radiated in the flash of bubble collapse, $E \sim \alpha\hbar c\frac{a^2}{c^3\tau^3}$. This leads to only 10 eV

for a flash of 1 fs. To get 10^7 eV, the normal energy of the flash, one needs a time scale as short as 10^{-17} s, if Larmor radiation alone is invoked. Milton noted that models are required in which the velocity of the radius is nonrelativistic while the acceleration becomes very large, and that this situation occurs during the formation of shocks.

3.5. Vacuum Radiation Effects and Squeezing in Fibers

Grishchuk *et al.* (1992), discussed a nonlinear Mach–Zehnder configuration to generate radiation through the optical squeezing of zero-point fluctuations interacting with a moving index grating. The squeezed vacuum optical radiation has been generated experimentally by separating the pump light from the squeezed fluctuations in the MZ interferometric geometry. In Section VIII of their paper the analogy with the vacuum particle production in cosmology is mentioned. There are two conditions to accomplish a strong optical squeezing in the laboratory: one needs to suppress classical noise and phase match the vacuum wave with the exciting source. These two conditions are very well satisfied by working with fibers.

3.6. Hyperbolic FS Vacuum Radiation and Channeling ($a \sim 10^{30} g_\oplus$, $T_\kappa \sim 10^{11}$ K?)

Relativistic particles can acquire extremely high transverse accelerations when they are channeled through crystals. Darbinian and collaborators (Darbinian *et al.*, 1989), related this physical setting to Unruh radiation but as before this is misleading and more reasonable is an interpretation in terms of a simplified Frenet–Serret trajectory. We include their interpretation for self-consistency reasons but one can figure that a FS approach to channeling could be worked out.

The idea is to detect the radiation emitted in the Compton scattering of the particles with the correlated spectrum of zero-point fluctuations of the crystal channel. The main argument is that the crystallographic fields are acting with large transverse accelerations on the channeled particles. The estimated transverse proper acceleration for positrons channeled in the (110) plane of a diamond crystal is $a = 10^{25}$ cm/s^2, and at $\gamma = 10^8$ one could reach 10^{33} cm/s$^2 = 10^{30} g_\oplus$. Working first in the particle instantaneous rest frame, the Erevan group derived the spectral angular distribution of the vacuum photons in that frame. By Lorentz transformation to the lab system they got the number of vacuum photons per unit length of crystal and averaged over the channeling diameter. At about $\gamma = 10^8$ the vacuum radiation intensity, defined as the intensity per unit pathlength of the Compton scattering on the Planck vacuum spectrum, becomes comparable with the Bethe–Heitler bremsstrahlung ($dN_\gamma/dE \propto 1/E$, and mean polar emission angle $\theta = 1/\gamma$).

Incidentally, this is in parallel with some experiments (Behning, 1990; Bini *et al.*, 1991; Unauthored, 1991), performed at LEP, where the scattering of the LEP beam from the thermal photon background in the beam pipe has been measured (the black body photons emitted by the walls of the pipe have a mean energy of 0.07 eV). It is considered as a fortunate case that the effect is too small to affect the lifetime of the stored beams.

3.7. FS Hyperbolic Vacuum Radiation From TeV Electrons in External Fields

In another work of the armenian group (Darbinian *et al.*, 1999), the same type of calculations was applied to get an estimate of the a *FS hyperbolic vacuum radiation generated by TeV electrons in a uniform magnetic field as well as in a laser field*". The FS vacuum radiation becomes predominant over the synchrotron radiation only when $\gamma = 10^9$ for $H = 5 \times 10^7$ Gs and consequently it is practically impossible to detect it at present linear colliders. Supercolliders with bunch structure capable of producing magnetic fields of the order 10^9 G are required. Pulsar magnetospheres are good candidates for considering such vacuum radiation patterns.

A circularly polarized laser field seems more promising since in this case the vacuum radiation could be detectable at lower magnetic fields and energies ($\gamma = 10^7$). This is due to the fact that the proper centripetal acceleration of the electron is $a = 2\omega\gamma\eta\sqrt{1 + \eta^2}$, where ω is the frequency of the electromagnetic wave, and $\eta = e\epsilon/m\omega$ (ϵ being the amplitude of the field).

3.8. FS Hyperbolic Vacuum Radiation and Ultraintense Lasers ($a \sim 10^{25}_{g_\oplus}$, $T_\kappa = 1.210^6$ K)

A very interesting proposal is to disentangle a Frenet–Serret vacuum radiation pattern in electron Petawatt-class laser interaction. It has been put forth by Chen and Tajima in 1999, who where motivated by the analogy with a "Unruh effect" (Chen and Tajima, 1999). In general terms, the probability of the photon emission depends on the autocorrelation function of the electromagnetic field along the "trajectory" of the electron. Uniform acceleration through the usual (Minkowski) quantum vacuum of the electromagnetic field distorts the two-point function of the zero-point fluctuations (ZPF) in the following way

$$\langle E_i(-\tau/2)E_j(+\tau/2)\rangle = \frac{4\hbar}{\pi c^3}\delta_{ij}\frac{(a/c)^4}{\sinh^4(a\tau/2c)}. \tag{75}$$

The main point of Tajima and Chen is to introduce the so-called laser strength parameter a_0 (see below) in this formula. In their approach in the leading order the accelerated electron is assumed "classical," with well-defined acceleration,

velocity, and position. This allows to introduce a Lorentz transformation so that the electron is described in its instantaneous proper frame. Also at this level the linearly accelerated electron will execute a classical Larmor radiation. As a response to the Larmor radiation, the electron reacts to the vacuum fluctuations with a nonrelativistic quivering motion in its proper frame that triggers additional radiation. The interaction Hamiltonian can be written as

$$H_{\text{int}} = -\frac{e}{mc}\vec{p}\cdot\vec{A} = -e\vec{x}\cdot\vec{E}. \tag{76}$$

The probability of the emission of a photon with energy $\omega = \epsilon' - \epsilon$ is

$$N(\omega) = \frac{1}{\hbar^2}\int d\sigma \int d\tau |\langle 1_{\vec{k}}, \epsilon'|H_{\text{int}}|\epsilon, 0\rangle|^2$$

$$= \frac{e^2}{\hbar^2}\sum_{i,j}^{3}\int d\sigma \int d\tau e^{-i\omega\tau}\langle x_i(\sigma)x_j(\sigma)\rangle\langle E_i(\sigma_-)E_j(\sigma_+)\rangle, \tag{77}$$

where $\sigma_- = \sigma - \tau/2$ and $\sigma_+ = \sigma + \tau/2$ are combinations of σ and τ, the absolute and relative proper time, respectively. The τ dependence of the position operator has been extracted to the phase due to a unitary transformation. The last bracket is the autocorrelation function for the fluctuations of the electric field in the vacuum.

The laser is treated as a plane EM wave and since the interest is in a *quasi-linear acceleration*, two identical, counterpropagating plane waves are considered in order to provide a standing wave

$$E_x = E_0[\cos(\omega_0 t - k_0 z) + \cos(\omega_0 t + k_0 z)], \tag{78}$$

$$B_y = E_0[\cos(\omega_0 t - k_0 z) - \cos(\omega_0 t + k_0 z)], \tag{79}$$

To get the quasi- linear acceleration and hopes for the Unruh signal, one should carefully tune the "laser" at the nodal points $k_0 z = 0, \pm 2\pi, \ldots$, where $B_y = 0$ identically for all times and E_x takes the maximum value. At $z = 0$, Chen and Tajima find

$$\gamma\beta_x = 2a_0\sin(\omega_0 t) \tag{80}$$

with

$$\gamma = \sqrt{1 + 4a_0^2 \sin^2 \omega_0 t}, \quad \text{and} \quad a_0 = \frac{eE_0}{mc\omega_0},$$

the relativistic γ factor and the dimensionless laser strength parameter, respectively.

The proper acceleration, which is related to that in the laboratory frame by $a = \gamma^3 a_{lab}$, is thus

$$a = 2ca_0\omega_0\cos\omega_0 t. \tag{81}$$

In order to calculate the autocorrelation function, one has to look for the transformation between laboratory and proper spacetimes. As $d\tau = dt/\gamma$, in the limit $a \gg 1$ one finds

$$\sin \omega_0 t = \tanh(2a_0\omega_0\tau)/\sqrt{1 + 4a_0^2 \text{sech}^2(2a_0\omega_0\tau)}$$

$$\sin k_0 x = 2a_0 \cos \omega_0 t / \sqrt{1 + 4a_0^2}. \tag{82}$$

For periodic motion, it is sufficient to examine the cycle $-\pi/2 \leq \omega_0 t \leq \pi/2 \rightarrow \pi/a_0 \leq \omega_0\tau \leq \pi/a_0$. Within the limit where $4a_0^2 \text{sech}^2(2a_0\omega_0\tau) \gg 1$, the above Equation reduces to

$$\sin \omega_0 t \approx \frac{1}{2a_0} \sinh(2a_0\omega_0\tau),$$

$$\cos k_0 x \approx \frac{1}{2a_0} \sinh(2a_0\omega_0\tau). \tag{83}$$

In the limit of small arguments, these can be readily recognized as conformal transformations of the Rindler transformation for constant acceleration. Thus, the experiment corresponds to a quasi-constant acceleration and therefore one can use the known formula (75).

$$\langle E_i(\sigma - \tau/2)E_j(\sigma + \tau/2)\rangle = \delta_{ij} \frac{4\hbar}{\pi c^3}(2a_0\omega)^4 \text{csch}^4(a_0\omega_0\tau). \tag{84}$$

From this, with some further mathematical tricks, one arrives at the basic relationships

$$\frac{dN(\omega)}{d\sigma} = \frac{1}{2\pi} \frac{e^2}{\hbar c^3}(2a_0\omega_0)^3 \langle x^2\rangle \int_{-\infty}^{+\infty} ds\, e^{-is\omega/a_0\omega_0} \text{csch}^4(s - i\epsilon). \tag{85}$$

$$\frac{dI_\kappa}{d\sigma} = \int_{\omega_0}^{\infty} \hbar d\omega \frac{dN(\omega)}{d\sigma} \approx \frac{12}{\pi} \frac{r_e \hbar}{c}(a_0\omega_0)^3 \log(a_0/\pi). \tag{86}$$

The above result applies to an accelerated electron located exactly at $z = 0$. At the vicinity of this point, e.g., $k_0 z = \epsilon \ll 1$, there is a nonvanishing magnetic field $|B_y| = \epsilon|E_x|$, which induces a $\beta_z \approx O(\epsilon)$ in addition to the dominant β_x. Nevertheless, the proper acceleration is affected only to the order ϵ^2

$$a \approx 2c\omega_0 a_0[1 - O(\epsilon^2)].$$

For electrons farther away where the decrease of acceleration becomes more significant, both the Unruh and the background Larmor radiations will decrease much more rapidly due to their strong dependences on acceleration. Thus only the origin and its immediate surroundings are important within this proposal.

3.8.1. FS Hyperbolic Vacuum Radiation Versus Larmor Radiation

At the classical level, the same linear acceleration induces the Larmor radiation, which is a huge background radiation for the "Frenet–Serret vacuum signal." The photon $\vec{k}$ space of interest for detection is along the direction of acceleration where the Larmor radiation is the weakest. A basic assumption is that the two radiations can be treated as independent processes without interference. The total Larmor radiation power is

$$\frac{dI_L}{dt} = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\right) = \frac{8}{3}r_e mc a_0^2 \omega_0^2 \cos^2(\omega_0 t), \tag{87}$$

and thus the total energy radiated during each laser half-cycle is

$$\Delta I_L = \frac{4\pi}{3}r_e mc a_0^2 \omega_0.$$

On the other hand, the hyperbolic vacuum radiation is significant over a reduced proper time period

$$\omega_0 \Delta\sigma \geq O(1/a_0)$$

Nevertheless, within this time the electron has become relativistic, with $\gamma \sim a_0$. As a result, the total energy radiated in the lab frame, i.e., $\Delta I_\kappa \sim (dI_\kappa)/(d\sigma)\gamma\,\Delta\sigma$, is $\Delta I_\kappa \sim \frac{12}{\pi}(r_e\hbar/c)a_0^3\omega_0^2\log(a_0/\pi)$. Thus the relative yield is

$$\frac{\Delta I_\kappa}{\Delta I_L} \sim \frac{9}{\pi^2}\frac{\hbar\omega_0}{mc^2}a_0\log(a_0/\pi). \tag{88}$$

Since $a_0 \propto 1/\omega_0$, the relative yield is not sensitive to the laser frequency. Chen and Tajima provided the following estimates of the relative yield

1. *Petawatt-class laser.* $\omega_0 \sim 2\cdot10^{15}\text{ sec}^{-1}$ and $a_0 \sim 100 \to \frac{\Delta I_\kappa}{\Delta I_L} \sim 3\cdot 10^{-4}$

2. *Free-electron laser-driven coherent X-ray source*: $\hbar\omega_0 \sim 10\,\text{keV}$ and $a_0 \sim 10 \to \frac{\Delta I_\kappa}{\Delta I_L} \sim 1$

3.8.2. Blind Spot of Larmor Radiation

The Frenet–Serret "thermal fluctuations" are considered isotropic in the electron's proper frame resulting in an isotropic induced radiation signal in the same frame. But since at each half cycle the electron rapidly becomes relativistic, with $\gamma \sim a_0$, the Unruh radiation is boosted along the direction of polarization (x axis) in the lab frame. Furthermore, as we have discussed above, the autocorrelation function, and therefore the FS signal, tend to diminish more rapidly than that from Larmor within the laser half cycle. This should induce a sharper time structure for

the former. Transforming the FS vacuum radiation power back to the lab frame with $\gamma \sim a_0$, the angular distribution in the small angle expansion becomes

$$\frac{d^2 I_\kappa}{dt\, d\Omega} = \frac{4}{\pi^2} \frac{r_e \hbar}{c} \frac{\omega_0^3 a_0^3}{[1 + a_0^2 \theta^2]^3}. \tag{89}$$

The Larmor radiation is polarized and its angular distribution in the small (θ, ϕ) polar-angle expansion is

$$\frac{d^2 I_L}{dt\, d\Omega} = \frac{8 r_e m c a_0^2 \omega_0^2}{[1 + a_0^2 \theta^2]^3} \left[1 - \frac{4 a_0^2 \theta^2 (1 - \phi^2)}{[1 + a_0^2 \theta^2]^2} \right]. \tag{90}$$

The Larmor power is minimum at $(\theta, \phi) = (1/a_0 \pi, 0)$, where the angular distribution is zero. Consider a detector which covers an azimuthal angle $\Delta\phi = 10^{-3}$ around the "blind spot," and an opening polar angle, $\Delta\theta \ll 1/a_0$. Then the partial radiation power for the FS signal would dominate over that for the Larmor within this solid angle. A plot confirming this *blind spot* has been drawn by Rosu (1999), and is displayed in Fig. 1.

On the other hand, Visser (2001), gave some interesting comments on the proposal of Chen and Tajima. I transcript here the following rethoric questions of Visser that he present as a sociology/linguistic issue:

> If you ultimately succeed in seeing this zero-point-fluctuations-induced modification to Larmor radiation, should you really call it the Unruh effect? Or should you just call it basic quantum field theory? After all you are not directly measuring the Unruh temperature itself.

3.8.3. How to Create Ultra-low Electrons?

Another important issue for this proposal is how to create ultra-low electrons at the nodes of the laser pulse. The suggestions of Chen and Tajima are the following.
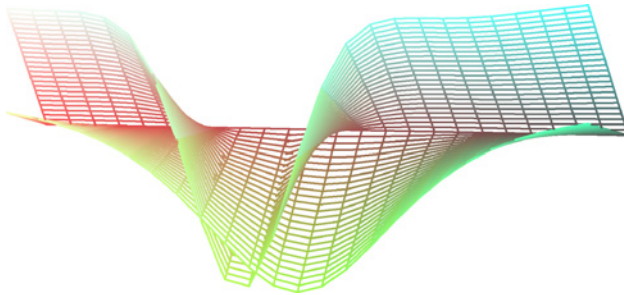

Fig. 1. The blind spot of Larmor radiation.

1. *Low energy photoelectrons* near the surface of a solid material.
2. *Laser-trapped and cooled electrons*. Laser-electron interactions occur in vacuum and there should be minimal additional background effects.
3. *Low temperature plasma*. The main background effect is the bremsstrahlung and to have a significant FS vacuum signal the plasma density should be low enough. The cross section of bremsstrahlung for an unscreened hydrogen nucleus per unit photon energy is

$$d\chi/\hbar\omega \sim \frac{16}{3}\alpha r_e^2 \ln(EE'/mc^2\omega).$$

The bremsstrahlung yield depends quadratically, whereas the FS signal linearly on the plasma density. The figure suggested by Chen and Tajima is $n_p \leq 10^{18} \mathrm{cm}^{-3}$ below which the bremsstrahlung background can be minimized.

3.9. Acceleration Temperature Formula and Damping in a Linear Focusing Channel

McDonald (1999), applied recently the "Unruh temperature formula" (equivalently, the "Frenet–Serret hyperbolic temperature formula") for a rapid calculation of the damping in a linear focusing channel [LFC].[2] He used the same idea some 15 years ago to reproduce Sands' results on the limits of damping of the phase volume of beams in electron storage rings.

3.10. Vacuum Acceleration Radiation in Quantum Optics (*Moderate Accelerations Could Work!?*)

This is the last proposal I am aware of and belongs to a Scully collaboration (Scully *et al.*, 2003). The idea is to enhance the vacuum acceleration signal from an accelerated He^+ ion used as a two-level detector of internal transition frequency ω passing through a high Q "single mode" cavity of frequency ν in the vicinity of the atomic frequency ω. The enhancement could be very significant leading to an effective Boltzmann factor linear in the parameter $\alpha/2\pi\omega$, where $\alpha = a/c$. By means of quantum optics calculations it can be shown that this type of thermal effect is due to nonadiabatic transitions stemming from the counter-rotating term $\hat{a}_k^+\hat{\sigma}^+$ in the time-dependent atom-field interaction Hamiltonian. The Larmor radiation lobes ($\propto \sin^2\theta$) are certainly present, but the blind spot in the forward direction of motion could be employed for the possible detection of this nonadiabatic effect.

---

[2] The LFC is a transport system that confines the motion of charged particles along straight central rays by means of a potential quadratic in the transverse spatial coordinates.

4. RELATED TOPICS

4.1. Unruh Effect and the Anomalous Doppler Effect (ADE)

When studied with the detector method, the FS vacuum radiation effect for a detector with internal degrees of freedom is very close to the anomalous Doppler effect (ADE). This is because in both cases the quantum detector is radiating "photons" while passing onto the upper level and not on the lower one (Frolov and Ginzburg, 1986), as shown in Fig. 2. It is worthwhile to note that the ADE-like concept was used by Unruh and Wald (Unruh and Wald, 1984; Higuchi *et al.*, 1992a,b), without referring to it explicitly, when they considered the Unruh effect for a uniformly accelerated quantum detector looked upon from the inertial reference frame. Their main and well-known conclusion was that emission in an inertial frame corresponds to absorption from the Unruh's "heat bath" in the accelerated frame. Essentially one may say the following.

(i) For the observer placed in the noninertial frame the "photon" is unobservable (it belongs to the left wedge in the Rindler case).

(ii) When the observer places himself in an inertial reference frame, he is able to observe both the excited quantum detector (furnishing at the same time energy to it) and the "photons." By writing down the energy–momentum conservation law he will be inclined to say that the "photons" are emitted precisely when the detector is excited.

4.2. Brief Review of the ADE Literature

A quantum derivation of the formula for the Doppler effect in a medium has been given by Ginzburg and Frank already in 1947 (Ginzburg and Frank, 1947),
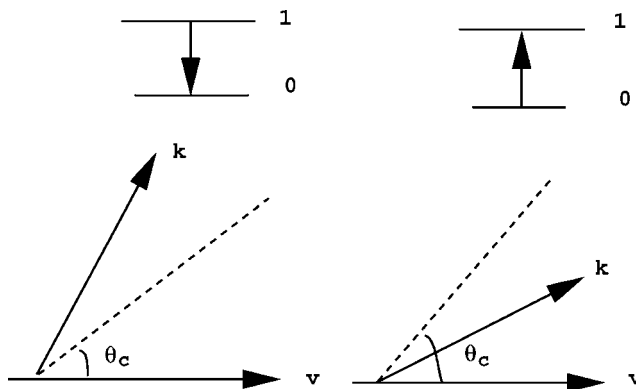

Fig. 2. The normal and anomalous Doppler effects and the corresponding transitions.

and more detailed discussion has been provided by Frank during 1970–1980 (Frank, 1979). See also the 1993 review paper of Ginzburg (Ginzburg, 1993).

Neglecting recoil, absorption, and dispersion (a completely ideal case) the elementary radiation events for a two-level detector with the change of the detector proper energy denoted by $\delta\epsilon$ are classified according to the photon energy formula (Frolov and Ginzburg, 1986),

$$\hbar\omega = -\frac{\delta\epsilon}{D\gamma} \tag{91}$$

where γ is the relativistic velocity factor ($\gamma > 1$) and D is the Doppler directivity factor

$$D = 1 - (vn/c)\cos\theta \tag{92}$$

The discussion of signs in Eq. (91) implies three cases as follows:

$D > 0$ for normal Doppler effect (NDE, $\delta\epsilon < 0$)

$D = 0$ for Cherenkov effect (CE, $\delta\epsilon = 0$, undetermined case)

$D < 0$ for anomalous Doppler effect (ADE, $\delta\epsilon > 0$).

Consequently, for a quantum system endowed with internal degrees of freedom the stationary population of levels is determined by the probability of radiation in the ADE and NDE regions. The possibility of doing population inversion by means of ADE has been tackled in the Russian literature since long ago. A quantum system with many levels propagating superluminally in a medium has been discussed for the first time by Ginzburg and Fain in 1958 (Ginzburg and Fain, 1958). The inverse population of levels by means of ADE or a combination of ADE and acceleration may be enhanced whenever the ADE region is made larger than the NDE region. This is possible, e.g., in a medium with a big index of refraction. Naryshkina (1962), already found in 1962 that the radiation of longitudinal waves in the ADE region is always greater than in the NDE region. However, her work remained unnoticed until 1984, when Nemtsov (1984), wrote a short note on the advantage of using ADE longitudinal waves to invert a quantum system propagating in an isotropic plasma. The same year, Nemtsov and Eidman (1984), demonstrated inverse population by ADE for the Landau levels of an electron beam propagating in a medium to which a constant magnetic field is applied. More recently, Kurian *et al.* (1988), have shown that in certain conditions (for certain range of the parameters), a detector moving with constant superluminous velocity on a circular trajectory inside a medium may be inverted too. Bolotovsky and Bykov (1989), have studied the space-time properties of ADE on the simple case of a superluminous dipole propagating in uniform rectilinear motion in a

nondispersive medium. These authors are positive with the separate observation of the ADE phenomenon for this case.

The radiation of a uniformly moving superluminal neutral polarizable particle has been studied by Meyer (1985). Frolov and Ginzburg (1986), remarked that this case is an analog of ADE due to zero-point fluctuations of electric polarizability.

Moreover, we can modify the index of refraction in the Doppler factor in such a manner as to get the ADE conditions already at sublight velocities. In this way a more direct link to the Unruh effect is available, as has been shown also by Brevik and Kolbenstvedt (1989). These authors studied in detail the DeWitt detector moving through a dielectric nondispersive medium with constant velocity as well as with constant acceleration. In this way, first order perturbation theory formulas for transition probabilities and rates of emitted energy were exploited.

4.3. Negative Energy Waves as Induced ADE Processes

Let us mention here that one way to look at negative energy waves in plasma physics is to consider them as a manifestation of induced ADE elementary events discussed in the book of Nezlin (1982). As a matter of fact, a number of authors have already dealt with the problem of amplification and generation of electromagnetic waves based on ADE in the field of quantum electronics (Ginzburg, 1979; Didenko *et al.*, 1983; Kuzelyev and Rukhadze, 1989). For details on the nonlinear instabilities in plasmas related to the existence of linear negative energy perturbations expressed in terms of specific creation and annihilation operators, and also for a discussion of the complete solution of the three-oscillator case with Cherry-like nonlinear coupling, one should consult the Trieste series of lectures delivered by Pfirsch (1991).

4.3.1. ADE Elementary Processes for Channeled Particles

Baryshevskii and Dubovskaya (1976), considered ADE processes for channeled positrons and electrons.

4.3.2. Negative Energy Waves in Astrophysics/Cosmology

Kandrup and O'Neill (1993), investigated the hamiltonian structure of the Vlasov–Maxwell system in curved background spacetime with Arnowitz–Desser–Misner splitting into space plus time, showing the importance of negative energy modes for time-independent equilibria.

4.4. Acceleration Vacuum Effects and Hadron Physics

The first application of the acceleration vacuum radiation concept to hadron physics belongs to Barshay and Troost (1978). These authors identified the hadronic Hagedorn temperature with the Unruh temperature (the Frenet–Serret

T_κ). In the same period, Hosoya (1979), and also Horibe (1979), applied moving mirror models to the thermal gluon production. An estimation of the contribution of thermal vacuum effects to the process of soft photon production by quarks is given in a paper by Darbinian *et al*. (1991). On the other hand, Dey *et al*. (1993), related the acceleration temperature to the observed departure from the Gottfried sum rule for the difference of the proton and neutron structure functions in deep inelastic electron scattering. Also, Parentani and Potting (1989), commented on the relationship between the limiting Hagedorn temperature, the maximal acceleration and the Hawking temperature.

It is worthwhile to append here the following remark. By coupling an Unruh–DeWitt detector with energy gap ΔE to a massive scalar field with mass $m > \Delta E$ the excitation per proper time of this detector when it is uniformly accelerated goes, for $\Delta E \ll m$, as

$$P/T \sim a \int_0^\infty dx x K_0^2 \left[\sqrt{x^2 + (m/a)^2} \right] \qquad (93)$$

This is clearly nonvanishing because the external agent is making work on the detector. On the other hand, the same detector lying inertial at rest in Minkowski spacetime in a thermal bath with temperature $T = a/2\pi$ will be just unable to excite because $\Delta E < m \le \omega$, where ω is the energy of the massive scalar particles. This fact does not challenge by any means the Unruh effect because what the Unruh effect does state is that Eq. (93) can be recovered in the uniformly accelerated frame by using Unruh's quantization plus the fact that the Minkowski vacuum is a thermal state of Unruh particles. This is so because the energy spectrum of the scalar particles in the Davies–Unruh thermal bath satisfies the constraint (despite the fact that they are massive) $\omega \ge 0$ rather than the usual $\omega \ge m$.

4.5. Decay of Accelerated Protons

The stability of protons has been long used as a basic test for the standard model of elementary particles. The most recent high-precision experiment set $\tau_p > 1.6 \times 10^{25}$ years (Caso *et al*., 1998), higher than the present age of the universe. However, long ago, Ginzburg and Syrovatskii (1965), speculated on the possibility of decay in the case of noninertially moving protons.

Recently, Vanzella and Matsas have calculated in the context of *standard QFT* (in inertial frames) the weak interaction decay rate for uniformly accelerated protons (Vanzella and Matsas, 2001),

$$p^+ \to n^0 + e_M^+ + \nu_M \qquad (94)$$

and shown that in certain astrophysical situations the proton lifetime may be quite short. The energy necessary to render the process (94) possible is supplied by the external accelerating agent. The tree-level formula for the proton proper lifetime in

1+1 spacetime dimensions (Matsas and Vanzella, 1999), (valid under the no-recoil condition $a \ll m_p$) is

$$\tau_{\text{Inert}}^{p \to n}(a) = \frac{2\pi^{3/2} e^{\pi \frac{\Delta m}{a}}}{G_F^2 m_e} \left[G_{1\ 3}^{3\ 0} \left(\frac{m_e^2}{a^2} \left| \begin{matrix} 1 \\ -\frac{1}{2}, \frac{1}{2} + i\frac{\Delta m}{a}, \frac{1}{2}, -i\frac{\Delta m}{a} \end{matrix} \right. \right) \right]^{-1}, \quad (95)$$

where G_{pq}^{mn} is the Meijer function (Gradshteyn and Ryzhik, 1980), a is the proton proper acceleration, $\Delta m \equiv m_n - m_p$ and $m_\nu = 0$ is assumed; m_p, m_n, m_e and m_ν are the rest masses of the proton, neutron, electron and neutrino, respectively. The effective Fermi constant $G_F = 9.9 \times 10^{-13}$ is determined from phenomenology.

Since a uniformly accelerated proton can be confined in a Rindler wedge which is a globally hyperbolic spacetime possessing a global time-like isometry, the associated uniformly accelerated observers (Rindler observers) must be able to analyze this phenomenon and reobtain the same (scalar) value for the proton lifetime (95) obtained with standard QFT. Notwithstanding, because of energy conservation, Rindler observers would simply claim that protons are precluded from decaying into a neutron through

$$p^+ \to n^0 + e_R^+ + \nu_R. \quad (96)$$

If Unruh's thermal bath (Minkowski vacuum) is taken into account new channels are opened:

$$p^+ + e_R^- \to n^0 + \nu_R \quad (97)$$

$$p^+ + \bar{\nu}_R \to n^0 + e_R^+ \quad (98)$$

$$p^+ + \bar{\nu}_R + e_R^- \to n^0, \quad (99)$$

i.e., from the point of view of the Rindler observers, the proton should be transformed into a neutron through the *absorption* of a Rindler electron and/or antineutrino from the surrounding thermal bath providing the necessary energy to allow the process to occur. Any energy in excess can eventually be disposed by the emission of a neutrino or a positron (depending on the case). Performing an independent QFT calculation in the uniformly accelerated frame, Vanzella and Matsas obtained the following proper lifetime for the proton, after combining (incoherently) processes (97)–(99) in the presence of the vacuum thermal bath:

$$\tau_{\text{Rindler}}^{p \to n} = \frac{\pi^2 a e^{\pi \frac{\Delta m}{a}}}{G_F^2 m_e} \left[\int_{-\infty}^{+\infty} d\omega_R \frac{K_{\frac{1}{2} + i\frac{\omega_R}{a}}(m_e/a) K_{-\frac{1}{2} + i\frac{\omega_R}{a}}(m_e/a)}{\cosh[\pi(\omega_R - \Delta m)/a]} \right]^{-1}. \quad (100)$$

Although Eqs. (95) and (100) appear to be quite different, Vanzella and Matsas have shown numerically that they coincide up to the level (Vanzella and Matsas, 2001),

$$\Delta^{-1} \int_\Delta \left[(\tau_{\text{Rindler}}^{p \to n} - \tau_{\text{inert}}^{p \to n})/\tau_{\text{Rinder}}^{p \to n} \right]^2 dx \sim 10^{-16},$$

where $x \equiv \log_{10}(a/1\,\text{MeV})$. This is an impressive result showing in a direct way that the Unruh interpretation is valid up to the sixteenth decimal figure.

5. CONCLUSIONS

The types of vacuum radiative effects that we have dealt here are in general manifestations of special quantum field vacuum states with spectral features that in some cases look similar but are of quite different origin with respect to the usual thermal thermodynamic effects. A key difference is, for example, that they are due to highly correlated states with EPR-type correlations (Pringle, 1989; Klyshko, 1991), and not due to a thermal uncorrelated state as the equilibrium states of statistical mechanics. Some of the most feasible detection proposals are related to nonadiabatic conditions, i.e., capable of producing very rapid oscillations, with the special pursuit of those cases in which the nonadiabaticity parameter depends on the proper acceleration. This allows an association with the Frenet–Serret vacuum radiations. A direct detection of the scalar vacuum radiation spectra has not been proposed so far. It requires the noninertial propagation of a source such as either a dislocation, a vortex, or a domain wall through the corresponding phonon medium.

ACKNOWLEDGMENT

REFERENCES

Abbott, L. F. and Wise, M. B. (1981). *American Journal of Physics* **49**, 37.
Artru, X. (1988). *Physics Letters A* **128**, 302.
Audretsch, J., Müller, R., and Holzmann, M. (1995). *Classical and Quantum Gravity* **12**, 2927.
Barber, D. P. and Mane, S. R. (1988). *Physical Review A* **37**, 456.
Barshay, S. and Troost, W. (1978). *Physics Letters B* **73**, 437.
Baryshevskii, V. G. and Dubovskaya, I. Ya. (1976). *Soviet Physics-Doklady* **21**, 741.
Bautista, E. (1993). *Physical Review D* **48**, 783.
Belkacem, A., Cue, N., and Kimball, J. C. (1985). *Physics Letters A* **111**, 86.
Belkacem, A., *et al*. (1986). *Physics Letters B* **177**, 211.
Bell, J. S. and Leinaas, J. M. (1983). *Nuclear Physics B* **212**, 131.
Bell, J. S., Hughes, R. J., and Leinaas, J. M. (1985). *Zeitschrift für Physik C* **28**, 75.
Bell, J. S. and Leinaas, J. M. (1987). *Nuclear Physics B* **284**, 488.
Bini, C., De Zorzi, G., Diambrini-Palazzi, G., Di Cosimo, G., Di Domenico, A., Gauzzi, P., and Zanello, D. (1991). *Physics Letters B* **262**, 135. .
Bini, D., Jantzen, R. T., and Merloni, A. (1999). *Classical and Quantum Gravity* **16**, 1.
Bolotovski, B. M. and Bykov, V. P. (1989). *Radiofizika* **32**, 386.
Brevik, I. and Kolbenstvedt, H. (1989). *Nuovo Cimento B* **103**, 45.
Brown, L. S. and Gabrielse, G. (1986). *Reviews of Modern Physics* **58**, 233.
Cai, Y. Q., Lloyd, D. G., and Papini, G. (1993). *Physics Letters A* **178**, 225.

Candelas, P. and Sciama, D. W. (1977). *Physical Review Letters* **38**, 1372.

Caso, C., *et al.* (1998). *European Physics Journal C* **3**, 1.

Caticha, A. (1992). *Physical Review B* **45**, 9541.

Chen, P. and Tajima, T. (1999). *Physical Review Letters* **83**, 256.

Cochran, W. (1989). *American Journal of Physics* **57**, 1039.

Cugliandolo, L. F., Kurchan, J., and Peliti, L. (1997). *Physical Review E* **55**, 3898 (condmat/9611044).

Darbinian, S. M., Ispirian, K. A., and Margarian, A. T. (1989). Preprint Yerevan Physics Institute YERPHY-1188(65)-89 (August 1989).

Darbinian, S. M., Ispirian, K. A., and Margarian, A. T. (1991). *Soviet Journal of Nuclear Physics* **54**, 364.

Darbinian, S. M., Ispirian, K. A., and Margarian, A. T. (1991). *Yadernaya Fizika* **54**, 600.

Darbinian, S. M., Ispiryan, K. A., Ispiryan, M. K., and Margaryan, A. T. (1990). *JETP Letters* **51**, 110.

Dehning, B., Melissinos, A. C., Perrone, F., Rizzo, C., von Holtey, G. (1990). *Physics Letters B* **249**, 145.

Denardo, G. and Percacci, R. (1978). *Nuovo Cimento B* **48**, 81.

Derbenev, Ya. S. and Kondratenko, A. M. (1973). *Soviet Physics-JETP* **37**, 968.

Dey, J., *et al.* (1993). *Physics Letters A* **172**, 203.

Didenko, A. N., *et al.* (1983). *Pis'ma v JTF* **9**, 1207.

Dodonov, V. V., Klimov, A. B., and Nikonov, D. E. (1993). *Physical Review A* **47**, 4422.

Dubovskaya, I. Ya., *et al.* (1993). *Journal of Physics: Condensation Matter* **5**, 7771.

Eberlein, C. (1996a). *Physical Review Letters* **76**, 3842.

Eberlein, C. (1996b). *Physical Review A* **53**, 2772.

Exartier, R. and Peliti, L. (2000). *European Physics Journal B* **16**, 119 (cond-mat)/9910412.

Feynman, R. and Vernon, F. (1963). *Annals of Physics* **24**, 118.

Frank, I. M. (1979). *Uspeki Fizicheskii Nauk* **129**, 685; *Vavilov-*Cherenkov Radiation. Theoretical Aspects*, Nauka, Moskow, 1988.

Frolov, V. and Novikov, I. (1993). *Physical Review D* **48**, 4545.

Frolov, V. P. and Ginzburg, V. L. (1986). *Physics Letters A* **116**, 423.

Ghose, P. and Home, D. (1994). *Physics Letters A* **191**, 362.

Ginzburg, N. S. (1979). *Radiofizika* **22**, 470.

Ginzburg, V. L. (1993). In E. Wolf, ed., *Progress in Optics XXXII*, Elsevier (Russian version in FIAN, Vol. 176, Nauka, Moskow, 1986). The 1993 version is updated and has a new section with comments on acceleration radiation.

Ginzburg, V. L. and Fain, V. M. (1958). *JETF* **35**, 817.

Ginzburg, V. L. and Frank, I. M. (1947). *Doklady Akademii Nauk* **56**, 583.

Ginzburg, V. L. and Syrovatskii, S. I. (1965). *Uspeki Fizicheskii Nauk* **87**, 65.

Gradshteyn, I. S. and Ryzhik, I. M. (1980). *Table of Integrals, Series and Products*, Academic Press, New York.

Grishchuk, L., Haus, H. A., and Bergman, K. (1992). *Physical Review D* **46**, 1440.

Hacyan, S. and Sarmiento, A. (1986). *Physics Letters B* **179**, 287.

Higuchi, A., Matsas, G. E. A., and Sudarsky, D. (1992a). *Physical Review D* **46**, 3450.

Higuchi, A., Matsas, G. E. A., and Sudarsky, D. (1992b). *Physical Review D* **45**, R3308.

Hizhnyakov, V. V. (1992). *Quantum Optics* **4**, 277.

Honig, E., Schucking, E. L., and Vishveshwara, C. V. (1974). *Journal of Mathematical Physics* **15**, 774.

Horibe, M. (1979). *Progress of Theoretical Physics* **61**, 661.

Hosoya, A. (1979). *Progress of Theoretical Physics* **61**, 280.

Hu, B. L. and Johnson, P. R. (2002). *QABP-2*, World Scientific, Singapore, quant-ph/0012132.

Iyer, B. R. and Vishveshwara, C. V. (1988). *Classical and Quantum Gravity* **5**, 961.

Iyer, B. R. and Vishveshwara, C. V. (1993). *Physical Review D* **48**, 5706, gr-qc/9310019.

Jadczyk, A. (1995). *Progress of Theoretical Physics* **93**, 631.

Jasper, S. J. (1947). *Bulletin of the École Polytechnical Jassy* **2**, 262.

Johnson, J. R., Prepost, R., Wiser, D. E., Murray, J. J., Schwitters, R. F., and Sinclair, C. K. (1993). *Nuclear Instruments and Methods* **204**, 261.

Kühne, M. (1989). *New Developments and Applications in Optical Radiometry*, Institute of Physics Conference Series No. 92, IOP Publishing Ltd.

Kandrup, H. E. and O'Neill, E. (1993). *Physical Review D* **48**, 4534.

Kats, Y., Kessler, D. A., and Rabin, Y. (2002). *Physical Review E* **65**, 020801.

Kim, S. K., Soh, K. S., and Yee, J. H. (1987). *Physical Review D* **35**, 557.

Kirsebom, K., *et al.* (2001). *Physical Review Letters* **87**, 054801.

Klein, A. and Landau, L. J. (1981). *Journal of Functional Analysis* **42**, 368.

Klyshko, D. N. (1991). *Physics Letters A* **154**, 433.

Korol, A. V., Solov'yov, A. V., and Greiner, W. (2002). *Journal of Physics G: Nuclear and Particle Physics* **28**, 627.

Kröger, H. (1997). *Physical Review A* **55**, 951.

Kurian, V. E., Pirojenko, A. V., and Florov, V. P. (1988). *On the Possibility of Population Inversion in Quantum Systems Moving Uniformly on a Circle in a Medium*, FIAN preprint 142; *KSF* **10**, 54.

Kuzelyev, M. V. and Rukhadze, A. A. (1989). *Fizika Plasmy* **15**, 1122.

Law, C. K. (1994). *Physical Review A* **49**, 433.

Lee, S. Y. (2000). *Accelerator Physics*, World Scientific, Singapore.

Leinaas, J. (1991). *Europhysics News* **22**, 78.

Letaw, J. R. (1981). *Physical Review D* **23**, 1709.

Letaw, J. R. and Pfautsch, J. D. (1980). *Physical Review D* **22**, 1345.

Letaw, J. R. and Pfautsch, J. D. (1981). *Physical Review D* **24**, 1491.

Levin, O., Peleg, Y., and Peres, A. (1993). *Journal of Physics A: Mathematical and General* **26**, 3001.

Man'ko, V. I. and Vilela Mendes, R. (1999). *Physics Letters A* **263**, 53 (physics/9712022).

Matsas, G. E. A. and Vanzella, D. A. T. (1999). *Physical Review D* **59**, 094004.

McDonald, K. T. (1987). In *Proceedings of the 1987 IEEE Particle Acceleration Conference*, Vol. 2, p. 1196.

McDonald, K. T. (1999). The Hawking-Unruh temperature and damping in a linear focusing channel. In *QABP 4-1*, World Scientific, Singapore, physics/0003061.

Meyer, P. P. (1985). *Journal of Physics A: Mathematical and General* **18**, 2235.

Milton, K. A., Comment on *Sonoluminescence as Quantum Vacuum Radiation*, quant-ph/9608003.

Moore, G. T. (1970). *Journal of Mathematical Physics* **11**, 2679.

Naryshkina, L. G. (1962). *JETF* **43**, 953.

Nemtsov, B. E. (1984). *Pis'ma v JTF* **10**, 588.

Nemtsov, B. E. and Eidman, V. Ya. (1984). *JETF* **87**, 1192.

Nezlin, M. V. (1982). *Dynamics of Beams in Plasmas*, in Russian, Energoizdat.

Parentani, R. and Potting, R. (1989). *Physical Review Letters* **63**, 945.

Pfirsch, D. (1991). *Negative-energy Modes in Collisionless Kinetik Theories and Their Possible Relation to Nonlinear Instabilities*, series of lectures at the Plasma College, ICTP-Trieste (June 1991).

Pringle, L. N. (1989). *Physical Review D* **39**, 2178.

Rogers, J. (1988). *Physical Review Letters* **61**, 2113.

Rosu, H. (1994). *Nuovo Cimento B* **109**, 423 and also gr-qc/9412033r.

Rosu, H. (1996). *Nuovo Cimento B* **111**, 507.

Rosu, H. (1999). *Physics World* **12**, 21, physics/9910015.

Rosu, H. C. (2001). *Gravitation and Cosmology* **7**, 1, gr-qc/9406012.

Rosu, H. C. (2002). *Artificial Black Holes*, World Scientific, Singapore, pp. 307–334, grqc/0012083.

Scully, M. O., Kocharovsky, V. V., Belyanin, A., Fry, E., and Capasso, F. (2003). *Physical Review Letters* **91**, 243004, quant-ph/0305178.

Sokolov, A. A. and Ternov, I. M. (1963). *Doklady Akademii Nauk* **153**, 1052–1054.

Sokolov, A. A. and Ternov, I. M. (1964). *Soviet Physics Doklady* **8**, 1203.

Sornette, D. (1990). *European Journal of Physics* **11**, 334.

Synge, J. L. (1967). *Proceeding of the Royal Irish Academy A* **65**, 27.

Tan, J. and Gabrielse, G. (1989). *Applied Physics Letters* **55**, 2144.

Tan, J. and Gabrielse, G. (1993) *Physical Review A* **48**, 3105.

Uggerhøj, U., private communication during QABP-3.

Unauthored (1991). *CERN Courier* **31**, 2.

Unruh, W. G. and Wald, R. M. (1984). *Physical Review D* **29**, 1047.

Vanzella, D. A. T. and Matsas, G. E. A. (2001). *Physical Review Letters* **87**, 151301.

Vanzella, D. A. T. and Matsas, G. E. A. (2001). *Physical Review D* **63**, 014010.

Visser, M. (2001). *Matters of Gravity*, Vol. 17, Spring 2001, gr-qc/0102044.

Yablonovitch, E. (1989). *Physical Review Letters* **62**, 1742.